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ON THE STABILIZATION OF UNSTABLE STEADY OPERATION MODE OF A DISPLACEMENT REACTOR WITH ALLOWANCE FOR HEAT RELEASE

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The problem of stabilizing an unstable optimal mode in a chemical reactor is solved by using a control system for the reagent supply proportional to the reactor temperature deviation. The effect of control parameters on stability is analyzed by the method of small perturbations. First, a model of a perfect control system is considered, and the minimum critical value of the stabilization parameter that ensures stability of the optimal reactor mode is determined. The numerical solution of the unsteady problem is used for establishing the criterion for selecting the stabilization parameter of the control system that would ensure stability of the reactor optimal mode. The effect of time lag in the control system on the feasibility of stabilizing an unstable mode is then examined. It is established that for a fixed stabilization parameter there exists a maximum time lag beyond which stabilization is impossible. The region of parameters that define the control system for which the optimal mode is stable, is determined.

Numerical solution of the unsteady problem is used for investigating the transition of a chemical reactor to the optimal mode at switching on the control system with various time lags. It is established that in the absence of time lag, i.e. in the case of perfect control system, the transition to a stable mode occurs without hunting. If the time lag is nonzero, the transition of the reactor to the stable state is accompanied by temperature and concentration profile fluctuations about their stable values in the optimal mode. These fluctuations become attenuated with time, and their amplitudes depend on the time lag. It is shown that for certain parameter values of the control system undamped fluctuations may occur in the reactor.

The model of a perfect displacement chemical reactor with allowance for the overall heat release [1], used for describing catalytic reactors particularly those with fluidized beds [2-6], can have three steady modes, viz. low, intermediate, and high. In the low steady mode the reactor temperature is too low for an efficient progress of the reaction. In the high mode the temperature is so high that decomposition of products and secondary reactions may often occur. Because of this the steady intermediate mode is the most suitable for operating chemico-technological processes. Unfortunately that mode is unstable [1], and one is faced with the problem of its stabilization.

The problem of stabilizing the unstable steady mode for the model of a complete intermixing reactor was considered in [7-11], where the control was by means of varying the flow rate of cooling fluid depending on temperature or concentration deviation from their steady values. Analysis shows that control by concentration does not always lead to stability, and that temperature is a more convenient variable for the control,

provided that the time lag is not excessive. Stabilization of an unstable mode using the model of a reactor with distributed parameters was, apparently, first considered in [12]. Since a zero order reaction was considered there, it was possible to restrict the analysis to that of the equation of temperature. It was found that any nonzero time lag in the control system makes stabilization of an unstable mode impossible.

1. Equations. Steady states and modes. Equations, boundary and initial conditions for defining the unsteady variations of temperature and concentration in the considered reactor model [1] can be presented in the dimensionless form as

$$\frac{\partial \xi}{\partial \tau} + v\frac{\partial \xi}{\partial x} = (1-\xi)\, g \exp\left(-\frac{\beta}{\theta}\right) \tag{1.1}$$

$$\frac{d\theta}{d\tau} = \alpha(\theta_0' - \theta) + \omega v(\theta_0'' - \theta) + \omega g \exp\left(-\frac{\beta}{\theta}\right)\int_0^1 (1-\xi)\, dx \tag{1.2}$$

$$x = 0, \quad \xi(0, \tau) = 0 \tag{1.3}$$

$$\tau = 0, \quad \xi(x, 0) = \xi_0(x), \quad \theta(0) = \theta_0 \tag{1.4}$$

$$\xi = \frac{c_0 - c}{c_0}, \quad x = \frac{X}{L}, \quad = \frac{T}{T^*}, \quad T^* = \frac{hc_0}{C_g\rho_g}$$

$$\tau = \frac{t}{\tau^*}, \quad \tau^* = \frac{L}{u^*}, \quad v = \frac{u}{u^*}, \quad \beta = \frac{E}{RT^*}, \quad g = k_0\tau^*$$

$$\alpha = \alpha_s \frac{S\tau^*}{V\rho_* c_*}, \quad \omega = \frac{\varepsilon_0\rho_g C_g}{\rho_* C_*}$$

k_0 is the pre-exponential multiplier

$$\rho_* C_* = \varepsilon_0\rho_g C_g + (1-\varepsilon_0)\rho_s C_s$$

In the above formulas X is the space coordinate $(0 \leqslant X \leqslant L)$; L is the reactor length; c is the concentration of key substance and c_0 its concentration at the reactor intake; ξ is the degree of the reaction advance; u is the reagent feed rate; ε_0 is the volume portion of reagent and reaction products of the catalyst porous layer; T is the temperature in the reactor; V and S are, respectively, the volume and lateral surface area of the catalyst; ρ_g and C_g are the density and specific heat of the mixture; ρ_s and C_s are the density and specific heat of the catalyst; α_s is the coefficient of heat transfer to the reactor lateral walls; T_0' is the temperature of ambient medium; T_0'' is the temperature of incoming mixture; h is the heat of reaction; E is the activation energy; R is the universal gas constant; u^* is the characteristic rate, and t is the time.

Equations (1.1) and (1.2) are based on the assumptions that diffusion of the reacting component is insignificant in comparison with the convective heat transfer, and can be neglected, while thermal conductivity is high and the temperature in the reactor can be considered constant. It is assumed that the reaction taking place in the reactor is irreversible and exotermic of the first order, whose rate depends on temperature in accordance with the Arrhenius law. It is further assumed that the stream velocity distribution over the reactor cross section is uniform, and that the reactor process is adiabatic, i.e. that $\alpha = 0$.

For the steady distribution of the degree of reaction advance $\xi^\circ(x)$ with allowance for $\alpha = 0$ and steady state temperature θ° from (1.1)-(1.3) we obtain

$$\xi^\circ(x) = 1 - \exp\left[-x\frac{g}{v}\exp\left(-\frac{\beta}{\theta^\circ}\right)\right] \tag{1.5}$$

$$\theta_0'' - \theta^\circ + 1 - \exp\left[-\frac{g}{v}\exp\left(-\frac{\beta}{\theta^\circ}\right)\right] = 0 \qquad (1.6)$$

Analysis of Eq. (1.6) shows that depending on θ_0'', β, g and v it can have from one to three solutions [1].

The typical form of dependence of θ° on v is shown in Fig. 1 (curve *1*) for $\theta_0'' = 1.75$, $\beta = 50$, and $g = \exp(25)$. Curve *1* intersects the vertical line $v = v_0$ at three points: O^-, O, and O^+ which correspond to the low, intermediate, and high temperature modes, respectively, at temperatures θ_1°, θ_2°, and θ_3. Of these modes the intermediate one is unstable.

Let us investigate the feasibility of stabilizing an intermediate mode by varying parameter v (the reacting mixture feed rate) in conformity with temperature and concentration deviation in the reactor from their steady state.

2. Control of the intermediate mode by temperature. We consider a reactor which has three steady modes (Fig. 1) with fixed parameters θ_0'', β, g, and $v = v_0$: We assume that the reactor comprises a system which makes it possible to vary the reacting mixture feed rate to it in proportion to temperature deviation in the reactor from its value in the intermediate steady state. We then have to set in Eqs. (1.1) and (1.2)

$$v(\tau) = v_0\{1 + d[\theta(\tau - \tau_d) - \theta_2^\circ]\} \qquad (2.1)$$

where θ_2° is the intermediate steady mode temperature at $v = v$, τ_d is the time lag determined by the degree of the control system inertia, and d is the stabilization parameter.

In the steady case instead of (1.6) from (1.1), (1.2), and (2.1) we have

$$\theta_0'' - \theta^\circ + 1 - \exp\frac{-g\exp(-\beta/\theta^\circ)}{v_0[1 + d(\theta^\circ - \theta_2^\circ)]} = 0 \qquad (2.2)$$

One of the solutions of Eq. (2.2) is, as previously, $\theta^\circ = \theta_2^\circ$. Other solutions that determine the steady temperatures in addition to θ_2° depend now on the stabilization parameter d. That dependence is shown in Fig. 2. In the absence of control ($d = 0$) steady temperatures maintain previous values θ_1°, θ_2°, and θ_3°. When $d \neq 0$ steady temperatures additional to θ_2° are determined by the ordinates of the intersection points of curve *1* (see Fig. 1) with the straight line of slope $d^{-1}v_0^{-1}$ passing through point v_0, θ_2°. It shows that with increasing parameter d (straight lines *2*—*7*) the temperature of the high steady mode decreases, while that of the low increases. At some $d = d_c$ the low mode merges with the intermediate (straight line *4*). Further increase of d brings temperature θ_2° to the level of the low mode (straight line *5*). Finally, when $d > d_1$, Eq. (2.2) has only the single solution θ_2° which corresponds to the unique steady mode (straight line *7*).

From the condition of tangency of straight lines with slope $d^{-1}v_0^{-1}$ to curve *1* at points O and D it is possible to determine d_c and d_1, for example,

$$d_c = \frac{\beta}{\theta_2^{\circ 2}} - \frac{v_0}{g}\exp\left[\frac{\beta}{\theta_2^\circ} + \frac{g\exp(-\beta/\theta_2^\circ)}{v_0}\right] \qquad (2.3)$$

By analogy to an uncontrolled reactor it is possible to expect that stabilization of the intermediate mode is reached when the stabilization parameter $d > d_c$ and the

solution $\theta^\circ = \theta_2{}^\circ$ of Eq. (2.2) corresponds to the low steady state. To check this we investigate the stability of steady states at various values of d.

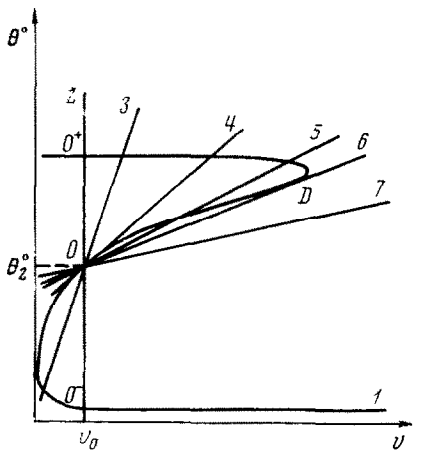

Fig. 1

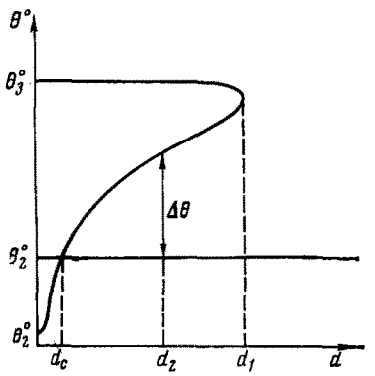

Fig. 2

Assuming that in (1.1) and (1.2)

$$\theta(\tau) = \theta^\circ + \theta'(\tau), \quad \xi(x, \tau) = \xi^\circ(x) + \xi'(x, \tau)$$

where θ° and $\xi^\circ(x)$ are the temperature and the degree of reaction advance in the steady mode, from (1.1)-(1.4) and (2.1) we obtain in linear approximation

$$\frac{\partial \xi'(x,\tau)}{\partial \tau} + v^\circ \frac{\partial \xi'(x,\tau)}{\partial x} + \frac{v_0}{v^\circ} b \exp\left(-\frac{b}{v^\circ}\right) d\theta'(\tau - \tau_d) - \frac{\beta b}{\theta^{\circ 2}} \exp\left(-\frac{b}{v^\circ}\right)\theta'(\tau) + b\xi'(x,\tau) = 0 \tag{2.4}$$

$$\frac{1}{\omega}\frac{d\theta(\tau)}{d\tau} + v^\circ\left\{1 - \frac{\beta}{\theta^{\circ 2}}\left[1 - \exp\left(-\frac{b}{v^\circ}\right)\right]\right\}\theta'(\tau) + v_0 d(\theta^\circ - \theta_0'')\theta'(\tau - \tau_d) + b\int_0^1 \xi'(x,\tau)dx = 0$$

$$b = g\exp\left(-\frac{\beta}{\theta^\circ}\right), \quad v^\circ = v_0[1 + d(\theta^\circ - \theta_2{}^\circ)]$$

$$x = 0, \quad \xi'(0,\tau) = 0$$

$$\tau = 0, \quad \xi'(x,0) = \xi_0'(x), \quad \theta'(0) = \theta_0$$

and by applying to this the Laplace transformation we obtain

$$p\xi'(x,p) - \xi'(x) + v^\circ \frac{d\xi'(x,p)}{dx} + b\exp\left(-\frac{b}{\theta^\circ}\right) \times \left[\frac{v_0}{v^\circ} d\exp(-p\tau_d) - \frac{\beta}{\theta^{\circ 2}}\right]\theta'(p) + b\xi'(x,p) = 0 \tag{2.5}$$

$$\frac{1}{\omega}[p\theta'(p) - \theta_0] + \left[v^\circ + v_0 d(\theta^\circ - \theta_0'')\exp(-p\tau_d) - \frac{\beta v^\circ}{\theta^{\circ 2}}\left(1 - \exp\left(-\frac{b}{v^\circ}\right)\right)\right]\theta'(p) + b\int_0^1 \xi'(x,p)\,dx = 0$$

$$x = 0, \quad \xi'(0,p) = 0$$

where p is the transformation parameter and all transforms are indicated everywhere by the argument p. The solutions of problem (2.5) for transforms of the degree of reaction advance and temperature are of the form

$$\xi'(x, p) = N(x, p)\exp\left(-\frac{p+b}{v^\circ}x\right) \tag{2.6}$$

$$\theta'(p) = (\theta_0 - \omega b D(p))\left\{p + M(p) + \frac{\omega b v^\circ}{p+b}\left(\frac{v_0}{v^\circ}d\exp(-p\tau_d) - \frac{b}{\theta^{\circ 2}}\right)\left[\frac{b}{p}\left(1 - \exp\left(-\frac{p}{v^\circ}\right)\right)\exp\left(-\frac{b}{v^\circ}\right) - 1 + \exp\left(-\frac{b}{v^\circ}\right)\right]\right\}^{-1}$$

$$N(x, p) = \int_0^x \left\{\exp\left(\frac{p+b}{v^\circ}z\right)\frac{\xi_0'(z) - M(p)\theta'(p)}{v^\circ}\right\}dz$$

$$\frac{M(p)}{\omega} = v^\circ + v_0 d(\theta^\circ - \theta_0'')\exp(-p\tau_d) - \frac{bv^\circ}{\theta^{\circ 2}}\left[1 - \exp\left(-\frac{b}{v^\circ}\right)\right]$$

$$D(p) = \frac{1}{p+b}\int_0^1 \xi_0'(x)\,dx - \frac{1}{p+b}\exp\left(\frac{p+b}{v^\circ}\right)\times \int_0^1 \xi_0'(x)\exp\left(\frac{p+b}{v^\circ}x\right)dx$$

All of the transforms singularities are poles, hence the problem of stability of the controlled steady mode reduces to the analysis of distribution of poles of functions $\xi'(x, p)$ and $\theta'(p)$ in the complex plane p. If all poles of these functions lie to the left of the imaginary axis, initial perturbations of the steady mode attenuate in time and the mode is stable but, if there are poles to the right of that axis, the mode is unstable.

The poles of transforms (2.6) lie at point $p = -b$ and at zeros of function

$$\Psi(s) = s^2 + a_1 s\exp(-sv^\circ\tau_d) + a_2 s + a_3\exp(-sv^\circ\tau_d) + a_4 + [a_5\exp(-sv^\circ\tau_d) - a_6]\times s^{-1}[1 - \exp(-s)],\quad s = p/v^\circ \tag{2.7}$$

where

$$a_1 = \frac{\omega v_0 d}{v^\circ}(\theta^\circ - \theta_0''),\quad a_2 = \omega - \frac{\omega\beta}{\theta^{\circ 2}}\left[1 - \exp\left(-\frac{b}{v^\circ}\right)\right] + \frac{b}{v^\circ}$$

$$a_3 = \frac{b\omega v_0 d}{v^{\circ 2}}\left[\theta^\circ - \theta_0'' - 1 + \exp\left(-\frac{b}{v^\circ}\right)\right],\quad a_4 = \frac{b\omega}{v^\circ}$$

$$a_5 = \frac{b^2\omega v_0 d}{v^{\circ 3}}\exp\left(-\frac{b}{v^\circ}\right),\quad a_6 = \frac{b^2\omega\beta}{v^{\circ 2}\theta^{\circ 2}}\exp\left(-\frac{b}{v^\circ}\right)$$

When considering the roots of equation $\Psi(s) = 0$ which have a positive real part we use the principle of argument according to which the number of zeros of function $\Psi(s)$ within a region bounded by the closed contour Γ along which $\Psi(s) \neq 0$, is equal to the increment of the argument of function $\Psi(s)$, when following round the contour, divided by 2π.

As the contour Γ we select $\Gamma_1 + \Gamma_2$ consisting of the right-hand semicircle of radius R with center at the coordinate origin (Γ_1) and of a segment of the coordinate axis lying between points $(0, R)$ and $(0, -R)$. When on the imaginary axis there are zeros of function $\Psi(s)$, contour Γ_2 is composed of segments of the ordinate axis and

right-hand semicircles of small radius r with centers located in the pure imaginary zeros of function $\Psi(s)$. We then pass to limit $R \to \infty$, $r \to 0$.

Analysis shows that the increment of argument of function $\Psi(s)$ along Γ_1 is independent of a and equal 2π when $R \to \infty$.

3. Perfect control. Let us investigate the case of perfect control with $\tau_d = 0$. Instead of (2.7) we have

$$\Psi(s) = s^2 + \Omega_3 s - \Omega_1 + \Omega_2 \frac{1 - e^{-s}}{s} \qquad (3.1)$$

$$\Omega_1 = -a_3 - a_4, \quad \Omega_2 = a_5 - a_6, \quad \Omega_3 = a_1 + a_2$$

Function (3.1) contains three parameters Ω_1, Ω_2, and Ω_3 which constitute a certain three-dimensional parametric space. Since these parameters are functions of the reactor characteristics and of the steady mode in which the reactor is working, hence to each of the steady states of a chemical reactor corresponds some point of that parametric space. The subsequent investigation is aimed at the determination in that parametric space of the stability region, i.e. a space where parameters Ω_1, Ω_2, and Ω_3 are such that function (3.1) has no roots with positive real parts. Then, when in the parametric space point (Ω_1^*, Ω_2^*, Ω_3^*) corresponds to some steady state of the reactor, that steady state is stable, and when the point lies outside that region, it relates to an unstable state. Let us determine the stability region of the parametric space.

Let us consider function (3.1) along contour Γ_2 where $s = iy$

$$\Psi(iy) = -y^2 - \Omega_1 + \Omega_2 \frac{\sin y}{y} + i\left(y\Omega_3 - \Omega_2 \frac{1 - \cos y}{y}\right) \qquad (3.2)$$

and determine parameters Ω_1, Ω_2, and Ω_3 for which function (3.1) has pure imaginary roots. It follows from (3.2) that this happens when the values of these parameters lie in the parametric space on a surface determined by equations

$$\Omega_1 = -y^2 + \Omega_3 \frac{y \sin y}{1 - \cos y}, \quad \Omega_2 = \Omega_3 \frac{y^2}{1 - \cos y} \quad (0 \leqslant y < \infty) \qquad (3.3)$$

The analysis of function (3.2) shows that in region $\Omega_1 > \Omega_2$ the increment of the argument of function (3.1) along contour Γ_2 when $R \to \infty$ is zero. Hence in that region function (3.1) has a root in the right-hand half-plane. The analysis has also shown that in the region comprised between surface (3.3) and the plane $\Omega_1 = \Omega_2$ the increment of the argument of function (3.1) along the contour Γ_2 is -2π, when $0 \leqslant y < 2\pi$. Hence in that region the increment of the argument of function (3.1) along the contour $\Gamma_1 + \Gamma_2$ is zero and function (3.1) has there no roots with a positive real part. This means that the region comprised between surface (3.3) and the plane $\Omega_1 = \Omega_2$ is the sought stability region.

The obtained data on the stability region in the parametric space $\Omega_1\Omega_2\Omega_3$ make possible the investigation of the effect of perfect control on the stability of the intermediate steady mode at temperature $\theta_2°$. Certain Ω_1^*, Ω_2^*, and Ω_3^* correspond to the intermediate steady mode in the absence of control ($d = 0$). Point $A = (\Omega_1^*, \Omega_2^*, \Omega_3^*)$ lies outside the stability region. With increasing parameter d the quantity Ω_1^* does not vary, while Ω_2^* and Ω_3^* increase, hence point A moves in the plane $\Omega_1 = \Omega_1^*$ toward the stability region whose dimensions increase. Changes in the relative po-

sition of the stability region and point A in the plane $\Omega_3 = \Omega_3^*$ are shown in Fig. 3 for three different values of the stabilization parameter d. Points A_1, A_2, and A_3 and the stability regions comprised between the straight line $\Omega_1 = \Omega_2$ and curves *1*, *2*, and *3*, correspond to $d_1 = 0$, $d_2 = 7.9$, and $d_3 = 12$, respectively. The stability region for $d = 0$ is shown shaded. It is seen that the intermediate mode becomes stable for finite values of the stability parameter d that exceed the critical value d_c at which the point reaches the stability region boundary, i.e. the straight line $\Omega_1 = \Omega_2$. It is not difficult to ascertain that the relation $\Omega_1^* = \Omega_2^*$ corresponds exactly to the equation derived in Sect. 2 for the determination of d_c. Hence the stabilization of the intermediate unstable mode occurs at its transition to the lower mode.

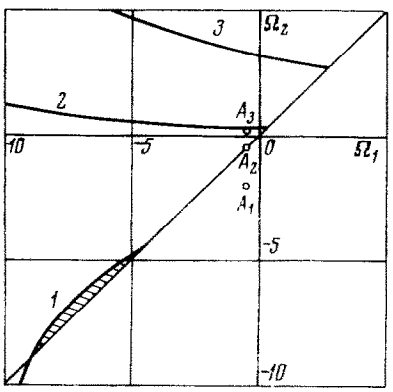

Fig. 3

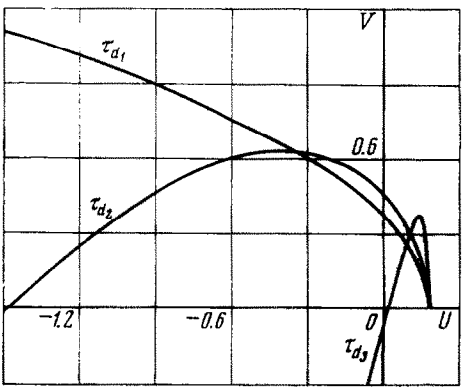

Fig. 4

The investigation of stability of the other two steady modes for various stabilization parameters is similar. It can be shown that the upper steady state remains stable up to its disappearance, i.e. until the stabilization parameter $d = d_1$ (see Fig. 2), while the lower steady state loses its stability when $d = d_c$, i.e. at passing to the unstable intermediate mode.

Note that decreasing parameter d beyond zero moves point A away from the stability region whose size then contracts. Hence it is not possible to achieve stabilization by selecting negative values for parameter d.

Let us consider the problem of selecting the stabilization parameter d. As previously stated, when $d > d_c$ the optimal temperature mode becomes stable. A numerical solution of the unsteady nonlinear system (1.1)-(1.4) had shown (see Sect. 5 below) that the maximum permissible level of finite perturbation increases with increasing stabilization parameter. For instance, (see Fig. 2) when $d = d_2$ the steady mode at temperature θ_2° is stable with respect to temperature perturbations of order $\Delta\theta$. When perturbations exceed the maximum permissible level, the reactor passes to the upper steady mode. The selection of a higher value for the stabilization parameter in the control system is, thus, preferable because the optimal steady mode under stabilization is stable with respect to perturbations of a larger scale.

4. Effect of time lag. Let us investigate the effect of the control system time lag τ_d on the stability of the mode that is being stabilized. We consider the case when the stabilization parameter $d > d_c$, so that with a perfect control system mode

$\theta^\circ = \theta_2^\circ$ is stable. We shall increase the time lag from zero and determine increments of the argument of function (2.7) along contour Γ_2 for each τ_d. The hodographs of vector $M = \Psi(iy) = V + iV$ $(0 \leqslant y)$ are plotted in Fig. 4 for several values of $\tau_{d_1} < \tau_{d_2} < \tau_{d_3}$. It will be seen that when $\tau_{d_1} = 0$ the increment of the argument of function (2.7) in the interval $(0, R)$ with $R \to \infty$ is equal $-\pi$ and, owing to the hodograph symmetry about the real axis, the total increment of the argument along contour Γ_2 is equal -2π. Consequently that increment along the complete contour Γ is zero when $\tau_{d_1} = 0$, and function $\Psi(s)$ has no zeros in the right-hand half-plane. This means that the considered steady mode is stable, which conforms to the results obtained earlier [1].

With increasing time lag the hodograph form changes. It appears that there exists a critical $\tau_d = \tau_d^*$, such that when $\tau_d > \tau_d^*$ the increment of the argument of function $\Psi(s)$ along contour Γ_2 is equal 2π, and the total increment is 4π. Hence a controlled steady mode becomes unstable when $\tau_d > \tau_d^*$. The hodograph which passes through point $(0, 0)$ corresponds to the critical time lag τ_d^*. From this condition by equating to zero the real and imaginary parts of function W we obtain

$$-y^2 + a_4 + [a_5 \cos(yv^\circ\tau_d^*) - a_6]\frac{\sin y}{y} - \tag{4.1}$$

$$a_5 \sin(yv^\circ\tau_d^*)\frac{1-\cos y}{y} + a_1 y \sin(yv^\circ\tau_d^*) + a_3 \cos(yv^\circ\tau_d^*) = 0$$

$$y[a_1 \cos(yv^\circ\tau_d^*) + a_2] + [a_6 - a_5 \cos(yv^\circ\tau_d^*)]\frac{1-\cos y}{y} -$$

$$\left[a_3 + a_5\frac{\sin y}{y}\right]\sin(yv^\circ\tau_d^*) = 0$$

The system of Eqs. (4.1) can be used for determining the critical value of the time lag τ_d^* as a function of the stabilization parameter d.

The results of numerical solution of system (4.1) are shown in Fig. 5. It is seen that the critical time lag diminishes with the increase of the stabilization parameter. The curve $\tau_d^* = \tau_d^*(d)$ divides there the complete region of parameters d and τ_d in two regions of which the shaded one contains values of the stabilization parameter and of time lag for which the stabilized steady mode is stable.

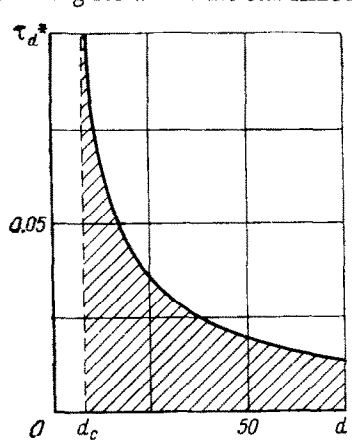

Fig. 5

For every time lag in the control system there exists an upper bound for the stabilization parameter, which ensures the stabilization of an unstable steady mode. Thus an excessively long time lag leads to the loss of stability.

We recall that it was established above that the instability of reactor parameters with related temperature perturbations in it impose a lower bound on the stabilization parameter of the control system. It is clear now that allowance for the time lag establishes the upper bound for the stabilization parameter. It is obvious that the greater the time lag in the control system, the lower must be the level of the reactor perturbation parameters if satisfactory stabilization of a steady mode is to be achieved.

Thus the investigations presented in this paper make it possible to formulate recommendations for determining control system parameters on the basis of characteristics of the considered reactor, which would ensure the stabilization of an unstable steady mode.

5. Results of numerical solution of the nonlinear problem. A numerical solution of problem (1.1)-(1.4) was obtained on a computer with the aim to investigate the unsteady behavior of a reactor and of its transition to a steady mode. Some of the results are shown in Fig. 6, where curves *1* and *2* illustrate transition of the reactor to a stabilized mode from the low steady state as the result of switching on of a proportional control at instant of time $\tau=0$. Curve *1* relates to the case when the stabilization parameter $d > d_c$ and the control are perfect, i.e. $\tau_d=0$. It is seen that the transition to the stable mode is monotonic.

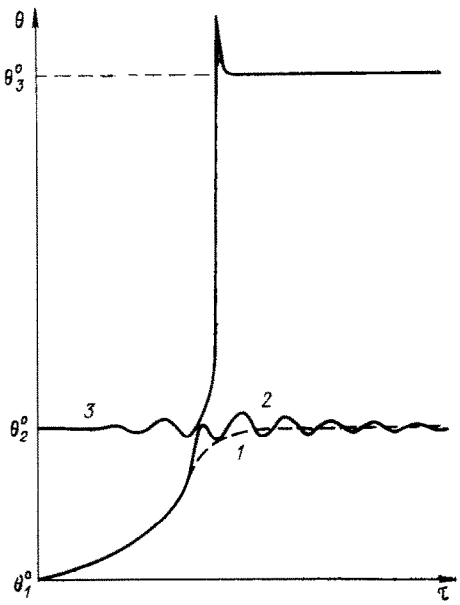

Fig. 6

Parameters d and τ_d for curve *2* are such that point (d, τ_d) lies in the shaded area of Fig. 5, which means, as previously stated, that the stabilized mode is stable. This shows that the presence of time lag induces fluctuations at transition to the stabilized mode. Numerical computations show that the amplitude of such damped fluctuations depends on parameters d and τ_d. Such temperature fluctuations in the reactor must be taken into account because of the possibility of overheating.

Curve *3* illustrates the reactor behavior when point (d, τ_d) lies outside the shaded area in Fig. 5. In that case the mode is unstable under stabilization. It can be shown by a method similar to that used in Sect. 4 that the time lag does not affect the stability of the high steady mode. It is seen from Fig. 6 that the reactor passes in that case from the controlled mode to the high steady state. If at the same time the stabilization parameter $d > d_1$ (Fig. 2), the high steady mode cannot exist and, as shown by numerical solutions on a computer, the reactor is subjected to undamped fluctuations.

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